W1 L9 - INTRO TO LINEAR DIFFERENTIAL EQUATIONS AND INTEGRATING FACTORS

Sep. $\frac{dy}{dx} = P(x) \cdot Q(x)$

$\Rightarrow \frac{1}{Q(y)} dy = P(x) dx$

Linear: $\frac{dy}{dx} = P(x) \cdot y + Q(x)$ ← linear

Idea: Write as $\underbrace{\frac{dy}{dx} + P(x) \cdot y}$ $= Q(x)$

treat as result of PRODUCT RULE
with a piece missing

$R(x) \cdot y = S(x)$

$R(x) \cdot \frac{d}{dx}(y) + R'(x) \cdot y = S'(x)$

$R(x) \frac{dy}{dx} + R'(x) \cdot y = S'(x)$

this is our missing piece

Divide everything by $R(x)$

We call our missing piece "$p(x)$"

$$p(x) \frac{dy}{dx} + \frac{R'(x) \cdot y}{R(x)} = \frac{S'(x)}{R(x)} \; p(x)$$

$$\frac{d}{dx}\left[e^{f(x)}\right] \rightarrow f'(x) e^{f(x)}$$

SO:

1. Get "y" + $\frac{dy}{dx}$ on ONE SIDE
2. Don't just integrate the left side
   - Find "missing" piece to create result of product rule

$$\frac{dy}{dx} + P(x) y = Q(x)$$

$$p(x) \frac{dy}{dx} + p(x) P(x) y = p(x) Q(x)$$

$\rightarrow p(x) \cdot y = \int p(x) Q(x) dx$

NEEDS:

1. Something that repeats itself: $e^{f(x)}$

$$\frac{d}{dx}\left[p(x)\right] = \frac{d}{dx}\left[e^{f(x)}\right] \rightarrow e^{f(x)} \cdot f'(x)$$

2. dy/dx is a deriv. w/ implicit differentiation
3. $P(x) = f'(x) \rightarrow f(x) = \int P(x) dx$

$\rightarrow p(x) = e^{\int P(x) dx}$

$$x^3 \cdot y = 2x$$

$\frac{dy}{dy}$

$$\frac{x^3 \cdot \frac{dy}{dx}}{x^3} + \frac{3x^2 \cdot y}{x^3} = \frac{2}{x^3}$$

$$\frac{dy}{dx} + \frac{3}{x}y = \frac{2}{x^3}$$

$P(x) = 3/x$

$$p(x) = e^{\int P(x)dx}$$

$$p(x) = e^{\int \frac{3}{x}dx} \Rightarrow p(x) = e^{3\ln x}$$

$$p(x) = e^{\ln x^3} = x^3$$

$$x^3\frac{dy}{dx} + x^3\,\frac{3}{x}y = x^3\,\frac{2}{x^3}$$

$$x^3\frac{dy}{dx} + 3x^2y = 2$$

$$\int D_x\left[x^3 \cdot y\right]dx = \int 2\,dx$$

$$x^3y = 2x$$

EX

$$\frac{dy}{dx} + y = 2, \quad y(0) = 0$$

1. Separable Var Method

$$\frac{dy}{dx} = (2-y)\cdot 1$$

$$\int \frac{1}{2-y}\,dy = \int 1\,dx$$

$$-\ln|2-y| = x + C$$

$$\ln|2-y| = -x - C$$

$$|2-y| = e^{-x-C}$$

$$2-y = \pm e^{-C}e^{-x}$$

$$2-y = C\cdot e^{-x}$$

$$2-0 = Ce^{-0} \rightarrow C = 2$$

$$2-y = 2e^{-x}$$

$$-y = -2 + 2e^{-x}$$

$$y(x) = 2 - 2e^{-x}$$

MATCH

2. Integrating Factors Method

$$\frac{dy}{dx} + 1y = 2$$

$$P(x) = 1$$

$$p(x) = e^{\int 1dx}$$

$$p(x) = e^x$$

you don't need a C when working with e^c in this method

$$e^x\frac{dy}{dx} + e^x y = 2e^x$$

$$\int D_x\left[e^x \cdot y\right]dx = \int 2e^x\,dx$$

it will be included here?

$$\Rightarrow e^x \cdot y = 2e^x + C$$

$$e^0 \cdot 0 = 2e^0 + C$$

$$-2 = C$$

$$\frac{e^x \cdot y}{e^x} = \frac{2e^x - 2}{e^x}$$

$$y = 2 - \frac{2}{e^x}$$